COMP 382: Reasoning about algorithms **Fall 2025**

Deadline: October 29, 2025 *Instructors: Aliakbarpour, Chida, Mamouras*

Homework 4 - Part a

Problem 1. (25 points)[Building a Business Portfolio]

Imagine you are an entrepreneur trying to build a business empire. Your goal is to maximize your **market influence** (MI), a score representing the number of successful ventures you control. Initially your MI is zero. You begin with a set amount of initial **capital** $C = C_0$ to invest. You are given a list of n **business opportunities**, each with a specific value v_i. For each business opportunity, **exactly one** of the two following operations can be executed:

- **Acquisition:** `acquire(i)`. This operation involves purchasing a new asset of value v_i. It can only be executed if your current capital is sufficient to meet the acquisition cost, $C \geq v_i$. Upon execution, your capital C is reduced by the asset's cost: $C \leftarrow C - v_i$. Your market influence (MI) increases by one point as the new venture is added to your portfolio: $MI \leftarrow MI + 1$.
- **Leverage:** `leverage(i)`. This operation is used to increase capital C. You can secure a loan of amount v_i by using one of your existing assets as collateral. This is only possible if you have at least one asset in your portfolio ($MI \geq 1$). Executing this gives you an immediate increase in your capital equal to v_i: $C \leftarrow C + v_i$. However, placing an asset as collateral causes market influence to decrease by one point: $MI \leftarrow MI - 1$.

Given an initial capital of C_0 and n business opportunities $V := \{v_1, \ldots, v_n\}$, design a greedy algorithm to come up with a **valid** business plan (a series of operations) where each business opportunity can be used at most once (either for acquisition or leverage, but not both). In your solution, provide the following:

(a) The greedy strategy and an overall step-by-step description of the algorithm.

(b) Pseudocode of your algorithm

(c) Details of implementation and analysis of the running time. Full credit will be given for algorithms that run in $O(n \log n)$ time.

(d) A proof of correctness for your algorithm.

Problem 2. (25 points) [Optimal Wireless Coverage]

A coastal path is represented by the positive real line, $(0, \infty)$. Along this path, there are n scenic trails, where each trail i is given as a closed interval $[x_i, y_i]$. For simplicity, assume that all x_i and y_i's are unique.

A wireless company offers the following contract: if we pay for trail $[x_k, y_k]$, they guarantee complete wireless coverage of that trail. Our goal is to pay for the smallest possible number of trails such that the following condition is satisfied: every trail either has coverage because we paid for it directly, or it overlaps with at least one trail that we did pay for.

Design a greedy algorithm that takes as input the collection of n intervals and outputs a set of trails that must be paid for with the minimum size. In your solution, provide the following:

(a) The greedy strategy and an overall step-by-step description of the algorithm.

(b) Pseudocode of your algorithm

(c) Details of implementation and analysis of the running time. Full credit will be given for algorithms that run in $O(n \log n)$ time.

(d) A proof of correctness for your algorithm.